Fast Kalman Processing of Carrier-Phase Signals from Global Navigation Satellite Systems for Water Vapor Tomography

Antti A. Lange

Finnish Meteorological Institute, P.O. Box 503, 00101 Helsinki, Finland
Email: antti.lange@fmi.fi

INTRODUCTION

An effective statistical inversion method is presented for adjusting both the space-geodetic measurements and the related meteorological observations simultaneously according to the best possible calibration standards [1]. The underlying joint regression equation system is first linearized and transformed into a Canonical Block-Angular (CBA) form. This is equivalent to the use of the Helmert-Wolf blocking method [2]. The generalized Canonical Correlation Analysis (gCCA) is the method of choice for those meteorological dependencies that can be modeled only statistically for real-time applications. The error covariance matrix of all estimated parameters is overwhelmingly large but it is computed and handled blockwisely using the same method. The theory of optimal Kalman Filtering applies as this exact error covariance matrix is computed operationally. Problems stemming from correlated measurement errors are overcome in near-real time with C. R.Rao's theory [3] of the Minimum Norm Quadratic Unbiased Estimation (MINQUE). It is computationally feasible through using a similar blocking method. All these computations are done extremely fast as the Normal Equations and the Quadratic system are solved semi-analytically exploiting Frobenius' simple inversion formula.

OPTIMAL KALMAN FILTERING

Equations (1) - (2) below describe the Kalman Filtering (KF) system used for the realtime fusion of all available data. Equation (1) tells how vector $\mathbf{y}_t$ of the measurements depends on vector $\mathbf{s}_t$ of the state parameters and on vector $\mathbf{e}_t$ of uncorrelated random errors at each epoch t. Vector $\mathbf{c}$ takes care of slow-varying systematic errors and calibration drifts. The linearized Measurement Equation (1) is thus as follows:

$$\mathbf{y}_t = H_t \, \mathbf{s}_t + F^y_t \, \mathbf{c} + \mathbf{e}_t \quad \text{for } t = 1, 2,... \tag{1}$$

where matrices H_t and F^y_t are basically the Jacobians that stem from partial derivatives of dependencies between the measurements, the state parameters and the systematic and calibration errors. Time evolution of the overall system is given by the linearized System Equation (2) as follows:

$$\mathbf{s}_t = A_t \, \mathbf{s}_{t-1} + B_t \, \mathbf{u}_{t-1} + F^s_t \, \mathbf{c} + \mathbf{a}_t \quad \text{for } t = 1, 2,... \tag{2}$$

where matrix A_t is the state transition matrix, B_t is the control gain matrix and F^s_t tells how the system depends on the slow-varying systematic errors and calibration drifts. Vector $\mathbf{s}_t$ of the present state of the overall system develops from its previous states $\mathbf{s}_{t-1}$ accordingl y but is also affected by some known controls $\mathbf{u}_{t-1}$ and process noise $\mathbf{a}_t$.

The measurement errors and process noise must not be auto- and cross- correlated less the optimality will be lost. Thus, these correlations are taken into account by modelling them in advance with the help of these matrices F^y_t and F^s_t. Such matrices are also found by statistical means using generalized Canonical Correlations techniques based on Singular Value Decomposition (SVD), see e.g. Lange (1999).

The Fast Kalman Filter (FKF) estimates of state vectors s_t, s_{t-1}, s_{t-2}, ... , s_{t-L-1} and all systematic errors c are computed using the Helmert-Wolf formulas:

$$s_{t-l} = \{X_{t-l}{}' V_{t-l}{}^{-1} X_{t-l}\}^{-1} X_{t-l}{}' V_{t-l}{}^{-1} (y_{t-l} - G_{t-l} c) \qquad (3)$$
$$c = \{\Sigma X_{t-l}{}' R_{t-l} X_{t-l}\}^{-1} \Sigma X_{t-l}{}' R_{t-l} y_{t-l}$$

where the summation index l (= 0, 1, 2, …, L-2, L-1) runs over sufficiently long time (=L) series of observational data so that all systematic and calibration error parameters c become estimable. The stability of this optimal Kalman filtering is controlled by monitoring the estimated error variances and by seeing that these stay within acceptable tolerances. For an explanation of the other formulas and symbols referred to, see [1].

ACCURACY ESTIMATION AND TOMOGRAPHY

The vector of all error variances of the GNSS signals is estimated by Rao's MINQUE method that takes the following simple form:

$$\hat{\sigma} = [\hat{\sigma}_1{}^2, \hat{\sigma}_2{}^2, ..., \hat{\sigma}_N{}^2]' = F^{-1} q \qquad (4)$$

where vector q is the vector of squared values of normalized carrier-phase residuals. F is a most tricky NxN matrix to be computed at each epoch. This is an obvious multivariate entension of estimating the accuracy of single mean value by computing first the standard deviation (=q) and then dividing it by the square-root of the number of the measurements (=F^{-1}). The Helmert-Wolf blocking method made it possible to perform these tricky computations in real-time. RAO's theory takes properly into account all mutual correlations between the different carrier-phase residuals.

A new program module collects these carrier-phase residuals from the C-files of MIT's GAMIT GPS Software and estimates the accuracy of each GPS signal at every epoch. Operational accurary of navigation becomes thus able to be assessed in real-time. Also the rapid Kalman estimates of the water vapor tomography and of the total electron content are expected improve internal consistency of these carrier-phases [4]. The national VRS network of dual-frequency GPS-receivers are connected with ADSL-lines, see Fig. 1 on the next page.

CONCLUSIONS

The horizontal accuracy and vertical resolution of water vapour tomography is improving due to the increasing number of signals from the GNSS satellites as well as of ever increasing density of the ground receiver networks. Operational quality control of each of the GNSS signals is obtained from their observed internal consistency using advanced statistical inference e.g. by the MINQUE theory [3]. The theory of optimal Kalman Filtering provides the most stable method for updating repeatedly various calibration and meteorological modeling parameters. The invented fast Kalman processing technique exploits effectively all data sources in real- and near-real time. Dependable accuracy information on all observed parameters are obtained for high-precision ultra-reliable navigation that improves control of all sorts of safety-critical vehicles [5].
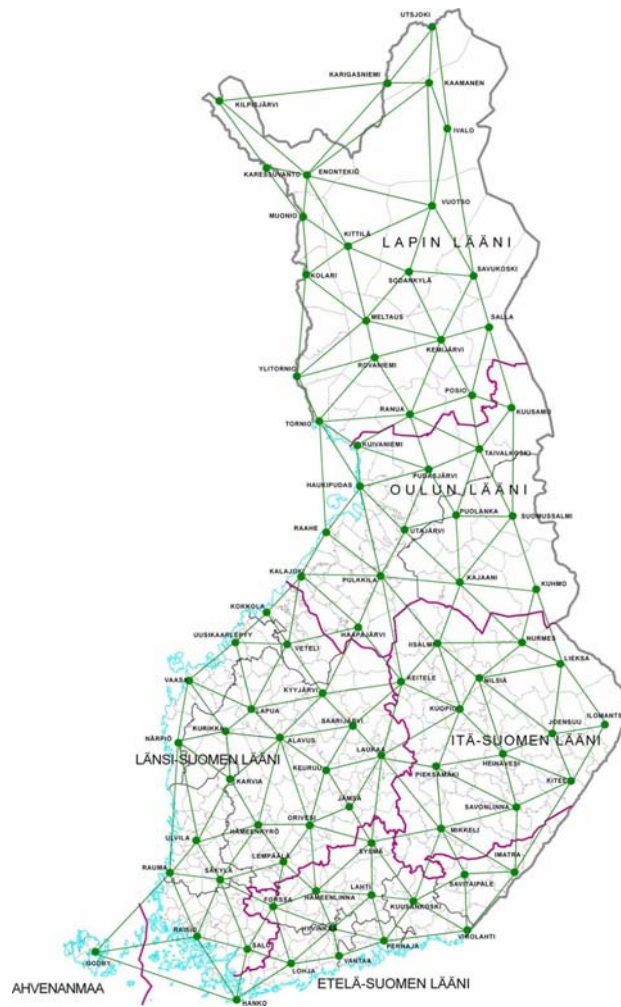
Figure 1. The national VRS network in Finland.

REFERENCES

[1] Lange, A. A., 2001. Simultaneous Statistical Calibration of the GPS signal delay measurements with related meteorological data. *Physics and Chemistry of the Earth*, Part A: Solid Earth and Geodesy, Vol. 26, No. 6-8, pp. 471-473.

[2] Wolf, H., 1978. The Helmert block method, its origin and development. In: Proceedings of the Second International Symposium on Problems Related to the Redifinition of North American Geodetic Networks, Arlington, Va.April 24-28, pp. 319-326.

[3] Rao, C. R., 1972. Estimation of variance and covariance components in linear models. *J. Am. Stat. Assoc.*, Vol. 67, No. 337, pp. 112-115.

[4] Nilsson, T., 2005. *Assessment of Tomographic Methods for Estimation of Atmospheric Water Vapor Using Ground-based GPS.* Academic Dissertation, Chalmers University of Technology, Göteborg, Sweden.

[5] Rizos, C., 1999. *Quality issues in real-time GPS positioning.* Report by Chairman, International Association of Geodesy SSG 1.154, IUGG Congress, Birmingham U.K., 18-29 July 1999.