MEASURING INTEGRITY OF NAVIGATION IN REAL-TIME

Antti A. Lange Ph.D.
Managing Director, FKF-Corporation Oy
Tilanhoitajankaari 18, 00790 Helsinki, Finland
+358-400-373182, www.FKF.net, lange@fkf.net

ABSTRACT

An ultra-reliable navigation receiver is an absolute necessity for the standardisation of modern cars, including the emerging driverless control. The receivers require an uncompromised level of integrity. The internal consistency of available signals from GNSS, pseudolites, beacons, radars, lidars, sonars, gyros, odometers, INS units, as well as all supporting data, is evaluated in a collective fashion. The Fast Kalman Filtering (FKF) method extends the Helmert-Wolf blocking (HWb) from Geodesy to precision navigation and solves in real-time the trickiest problem of calculating the Minimum Norm Quadratic Unbiased Estimates (MINQUE) of signal errors for the needs of safe driving. This patented FKF method makes full advantage of all possible combinations of the different signals, even in a numerically exploitable manner. Fully automated driving can then be made safe by using all available signals at reasonably restricted cruising speeds. This reported intelligent determination of the true precision of navigation makes room for smooth transport.

Keywords:
The Fast Kalman Filtering (FKF), Canonical Block-Angular (CBA), Helmert-Wolf blocking (HWb), Minimum Norm Quadratic Unbiased Estimation (MINQUE), Receiver Autonomous Integrity Monitoring (RAIM), Fault Detection and Exclusion (FDE), A Posteriori Multipath Estimator (APME).

INTRODUCTION

The Receiver Autonomous Integrity Monitoring (RAIM) has been used for safety-critical GNSS navigation. The Fault Detection and Exclusion (FDE) enabled reliable operation even in the presence of a signal failure. The differences between the expected and observed signal values are divided by their normal variation. These ratios are compared with a threshold value for a small probability of false alarms. The Best possible Linear Unbiased Estimation (BLUE) of the expected signal values requires extremely fast computing. The Real-Time Kinematic (RTK) and Virtual Reference Station (VRS) land surveying also need the best possible accuracy. These precision techniques exploit the fastest sparse-matrix method known as the Helmert-Wolf blocking (HWb). However, the most reliable estimates of these attained accuracies can only be calculated in late processing. This paper discloses the patented FKF method for making the precise computations operationally possible even in real-time so that the true accuracy of each signal is used for the RAIM , FDE and APME processing.

FAST KALMAN FILTERING (FKF)

Helmert-Wolf blocking (HWb)

The linearized joint regression equation system (1) of the signal data from all receivers and/or observing instruments can be written out in the Canonical Block-Angular (CBA) form as outlined first by F. R. Helmert in 1880:

$$\begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \vdots \\ \mathbf{y}_K \\ \underline{\mathbf{c}} \end{bmatrix} = \begin{bmatrix} X_1 & & & & G_1 \\ & X_2 & & & G_2 \\ & & \ddots & & \vdots \\ & & & X_K & G_K \\ & & & & I \end{bmatrix} \begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \\ \vdots \\ \mathbf{b}_K \\ \mathbf{c} \end{bmatrix} + \begin{bmatrix} \mathbf{e}_1 \\ \mathbf{e}_2 \\ \vdots \\ \mathbf{e}_K \\ \mathbf{e}_C \end{bmatrix}$$

i.e. (1)

$$\mathbf{y} \quad = \qquad \mathbf{H} \qquad \mathbf{s} \quad + \quad \mathbf{e}$$

where vectors $\mathbf{y} = [\mathbf{y}_1', \mathbf{y}_2', \ldots, \mathbf{y}_K', \mathbf{y}_{K+1}']'$, $\mathbf{s} = [\mathbf{b}_1', \mathbf{b}_2', \ldots, \mathbf{b}_K', \mathbf{c}']'$ and $\mathbf{e} = [\mathbf{e}_1', \mathbf{e}_2', \ldots, \mathbf{e}_K', \mathbf{e}_{K+1}']'$ represent the measurements $\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_K$ of *n* different signals for K consecutive locations and their a priori calibration data $\underline{\mathbf{c}}$, the unknown states $\mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_K$ for each of the K consecutive locations and the *m* unknown common calibration parameters **c** and the error vectors $\mathbf{e}_1, \mathbf{e}_2, \ldots, \mathbf{e}_K$ of the signals and the error vector $\mathbf{e}_C$ of the a priori calibration data, respectively, during a sampling period t (t = 1, 2,...) from all available data blocks k (k = 1, 2,..., K) so that:

$\mathbf{y}_k$ = vector of *n* signals

$\underline{\mathbf{c}} = \mathbf{y}_{K+1}$ = vector of the a priori values of the *m* common calibration parameters

X_k = Jacobian matrix for the separate state parameters $\mathbf{b}_k$
G_k = Jacobian matrix for the *m* common calibration parameters **c**
G_{K+1} = I = identity matrix
$\mathbf{b}_k$ = vector of the separate state parameters
c = vector of the *m* common calibration parameters

$\mathbf{e}_k$ = vector of the errors of the *n* signals $\mathbf{y}_k$

$\mathbf{e}_C = \mathbf{e}_{K+1}$ = vector of the errors of the a priori values of the *m* calibration parameters.

The state parameters $\mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_K$ are estimated simultaneously for each data block k. The unknown calibration parameters are more or less common to several data blocks k (k = 1, 2,..., K) and are denoted here by vector **c,** as they are typically slow-varying or almost constant.

Matrix $\mathbf{H}$ is typically an extremely huge Jacobian matrix, though it can be written out in the Canonical Block-Angular (CBA) form of equation (1) where its submatrices $X_1, X_2, \ldots, X_K$ and $G_1, G_2, \ldots, G_K, G_{K+1}$ are moderately sized Jacobians of the state and of the calibration parameters. Wolf's formulas for computing all these state and calibration parameters are as follows:

$$\mathbf{b}_k = (X_k' X_k)^{-1} X_k' (\mathbf{y}_k - G_k \mathbf{c}) \quad \text{for } k = 1, 2, \ldots, K;$$

and, (2)

$$\mathbf{c} = (\sum G_k' R_k G_k)^{-1} \sum G_k' R_k \mathbf{y}_k$$

where both indexes k of the two summations $\sum$ run over K+1 blocks; and,

where $R_k = I - X_k (X_k' X_k)^{-1} X_k'$ = residual operator for data block k; and,

$R_{K+1} = I$ = "residual operator" for the a priori calibration data $\mathbf{y}_{K+1}$;

$G_{K+1} = I$ = "Jacobian matrix" for the a priori calibration data $\mathbf{y}_{K+1}$;

The added value of K+1 for the index k above extends conventional Wolf's formulas (2) into a large optimal Kalman filter as disclosed in the FKF patent description. However, the absolutely Best Linear Unbiased Estimates (BLUE) of the state and the *m* calibration parameters are obtained only if all covariance matrices $Cov(\mathbf{e}_k) = E(\mathbf{e}_k \mathbf{e}_k')$ of the error vectors $\mathbf{e}_1, \mathbf{e}_2, \ldots, \mathbf{e}_K, \mathbf{e}_{K+1}$ are initially transformed into identity matrices in such a way that also $E(\mathbf{e}_{k1} \mathbf{e}_{k2}') = 0$ for all indexes $k1 \neq k2$. The true error covariance matrix of all the estimated parameters $\mathbf{s}$ will thereafter take the following most simple and also numerically exploitable form:

$$Cov(\mathbf{s} - E(\mathbf{s})) = E[\mathbf{s} - E(\mathbf{s})][\mathbf{s} - E(\mathbf{s})]' = \quad (3)$$

$$= \begin{bmatrix} C_1 + D_1SD_1' & D_1SD_2' & \cdots & D_1SD_K' & -D_1S \\ D_2SD_1' & C_2 + D_2SD_2' & & D_2SD_K' & -D_2S \\ \vdots & & \ddots & \vdots & \vdots \\ D_KSD_1' & D_KSD_2' & \cdots & C_K + D_KSD_K' & -D_KS \\ -SD_1' & -SD_2' & \cdots & -SD_K' & S \end{bmatrix}$$

where $S = (\sum G_k' R_k G_k)^{-1}$ the summation k runs over data blocks k=1, 2, … K, K+1; and,

$C_k = (X_k' X_k)^{-1}$ for k=1, 2, … K,

$D_k = (X_k' X_k)^{-1} X_k' G_k$ for k=1, 2, … K.

The required normalization of the error vectors $\mathbf{e}, \mathbf{e}_2, \ldots, \mathbf{e}_K, \mathbf{e}_{K+1}$ of the *n* signals and of the *m* calibration parameters has to be made by suitably transforming the measurement equations (1). Multivariate statistical methods such as a generalized Canonical Correlation Analysis (gCCA) may also be needed for the elimination of an unwanted heteroscedasticity from the error variances. However, so far it has been an overly difficult task to even estimate the error covariance matrix $Cov(\mathbf{e}) = E(\mathbf{e}\,\mathbf{e}')$ itself, not to mention its real-time computation for each sampling period t (t = 1, 2,...) for operational purposes.

Estimating the true Precision

C. R. Rao's (1972 and 1975) Minimum Norm Quadratic Unbiased Estimation (MINQUE) theory provides the only practical approach for an efficient estimation of the error covariance matrix Cov(**e**) = E(**e e**') of the various signals and their calibration data. However, the reliable mathematical solution calls for overwhelmingly large matrix inversions. No existing Floating-Point Unit (FPU) can deal with them due to short mantissas. Susan D. Horn, Roger A. Horn and David B. Duncan developed an Almost Unbiased Estimation (AUE, 1975) method from MINQUE though it is not always sufficient. Fortunately, the patented FKF method using the semi-analytic sparse-matrix techniques of HWb can usually solve these problems with many operational applications of navigation, mobile positioning, probe or vehicle tracking as well as Astro- and Geophysics.

Vector σ of the $n+m$ error variances of all the signal and calibration input data can now be estimated according to Rao's MINQUE theory – under an assumption of the so-called translation invariance – by employing the following simple-looking formula:

$$\sigma = [\sigma_1^2, \sigma_2^2, \ldots, \sigma_N^2]' = F^{-1}\mathbf{q} \tag{4}$$

where σ = vector of N unknown error variances

N = $n+m$ = total number of the n different signals and the m calibration inputs

$\mathbf{q}$ = vector of the N sums of squared computed residuals $\mathbf{e} = [\mathbf{e}_1', \mathbf{e}_2', \ldots, \mathbf{e}_K', \mathbf{e}_{K+1}']'$

F = NxN square matrix.

The residuals computed from the vector $\mathbf{y} = [\mathbf{y}_1', \mathbf{y}_2', \ldots, \mathbf{y}_K', \mathbf{y}_{K+1}']'$ are called the *innovation sequence* of Kalman filtering. The hardest task here is to compute the matrix F in real-time. Quadratic estimators like these error variances may, in principle, obtain negative values if strictly unbiased. Good estimates may also be obtained using an AUE approximation of the matrix F, and the AUE variances always turn out positive. These MINQUEs can be computed operationally by applying the FKF method as follows:

$$\mathbf{q} = [\,\mathbf{y}'\mathbf{RT}_1\mathbf{y}, \mathbf{y}'\mathbf{RT}_2\mathbf{y}, \mathbf{y}'\mathbf{RT}_3\mathbf{y}, \ldots, \mathbf{y}'\mathbf{RT}_N\mathbf{y}\,]' \tag{5}$$

F = matrix { trace $(\mathbf{RT}_i\,\mathbf{RT}_j)$} wherein i = 1, 2, 3,... , N and j = 1, 2, 3,... , N

$\mathbf{R} = \mathbf{I} - \mathbf{H}\ \mathrm{Cov}(\mathbf{s} - \mathrm{E}(\mathbf{s}))\ \mathbf{H}'$ and

$\mathbf{T}_i$ = gigantic sparse-matrix (i = 1, 2, 3,... , N).

The sparse matrix $\mathbf{T}_i$ is diagonal for a typical navigation application. Its scalar elements are Kronecker's deltas that indicate which data from one of the N different signals are selected

e.g. $\mathbf{T}_i = \mathrm{diag}\{\,\mathrm{diag}(\delta_{i,1}, \delta_{i,2}, \ldots, \delta_{i,h1}), \mathrm{diag}(\delta_{i,1}, \delta_{i,2}, \ldots, \delta_{i,h2}), \ldots, \mathrm{diag}(\delta_{i,1}, \delta_{i,2}, \ldots, \delta_{i,hK+1})\,\}$

where $\delta_{i,h} = 1$ if the place h in subdiagonal k (k=1,2,..., K+1) is entitled to the input data from the signal i; otherwise $\delta_{i,h} = 0$. Here we will have at least n*K+m such diagonal elements.

The CBA equations (1) should be normalized again and again using the improving new error variance estimates as follows:

$$\mathrm{Cov}(\mathbf{e}) = \sum \sigma_i^2 \mathbf{T}_i \quad \text{where the summation index i runs from 1 to N.} \qquad (6)$$

This kind of accuracy estimation does not, in the long run, depend essentially on the given a priori values as long as the FKF process is optimal in all other respects. Equations (3) - (6) will then provide objective accuracy estimates as they are derived from the real-time integrity of the entire system. In the trivial case of only one dimension, these formulas will reduce to a simple calculation of the error variance of a mean value.

CONCLUDING REMARKS

The theory of optimal Kalman filtering provides the only mathematically correct means for repeatedly updating receiver positions, instrumental calibration drifts, system model and environment parameters in the most stable way. Therefore, the FKF processing method exploits here large moving data windows which can be kept sufficiently long in order to satisfy Kalman's observability and controlabilty conditions also for all those calibration parameters that are involved in safety-critical navigation. The only straightforward way of controlling validity of these two most important conditions is to monitor the true accuracies by measuring them continuously in real-time as described above. However, no conventional optimal Kalman filter is able to deal with the emerging excessive computational burden because its numerical complexity is proportional to the cube of the number of the numerous input signals and variables. Fortunately, the numerical complexity of FKF is only roughly proportional to the square. Sophisticated A Posteriori Multipath Estimator (APME) techniques can thus be developed for full exploitation of the currently available numerous GNSS, position location and IMU signals. Consequently, the forthcoming ultra-reliable hybrid navigation receivers will improve both cruise controlling and high-precision piloting of all sorts of vehicles. Simulations demonstrate how combinations of many new data sources helps to improve navigation accuracy for more intelligent and safer transports.

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