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BALLOON TRACKING USING A HYBRID SYSTEM

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BALLOON TRACKING USING A HYBRID SYSTEM

Abstract

Geographical coverage of Loran-C navigation chains reaches out only to areas where clean groundwave pulses can be received at least from three transmitting stations. More distant Loran-C signals suffer from skywave interference but it seldom degrades accuracy of balloon tracking below requirements of Synoptic Meteorology. Thus, Loran-C signals are usable far beyond their navigational coverage. However, Geometric Dilution Of Precision (GDOP) often becomes so severe that it totally or occasionally prevents Loran-C from being used for upper-air windfinding.

The paper outlines how Loran-C signal data can be combined with angular measurements from Radio and/or Optical Theodolites. In fact, the use of only one or two Loran-C signals in combination of a theodolite transforms the theodolite into a hybrid system called “secondary radar.” It is the distance measuring capability of a radar that makes it perform remarkably well during strong wind conditions. Thus, combinations of different signals and sensors are exploited in maritime, aeronautical and space applications in order to improve GDOP. However, a hybrid system is often sensitive to inconsistencies caused by systematic errors in its subsystems. A minor calibration drift may create grossly misleading positional information. As upper-air windfinding uses differentiation/ differencing in time the computed winds can be even more distorted.

Balloon tracking experiments were made with a hybrid balloon tracking system that was temporarily assembled at Radiosonde Station 02935 in Finland in July 1997. The tracking data was collected from five consecutive soundings comprising Loran-C signals from the two chains of Ejde (9007) and Bryansk (8000) as well as angular measurements of an Optical Theodolite (OT) and a ME12 Radio Theodolite (RT). Importance of adequate calibration is demonstrated by using a statistical calibration method based on Kalman Filtering (KF). However, this leads to inversion of huge matrices where high-precision arithmetics of High-Performance Computing (HPC) would normally be required. The method of Fast Kalman Filtering (FKF) was used instead as it has now been implemented on a Pentium PC under title Hybrid Windfinding Algorithm (HWA). Its FORTRAN coded modules can be retrieved for testing from FTP server ftp.out.fmi.fi as ASCII files under user: *hwa* using password: *hwa*. However, the Finnish Meteorological Institute assumes no responsibility for any misuse or faults of the highly sophisticated pieces of software.

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1. INTRODUCTION

A general mathematical model for meteorological balloon tracking was formulated by the author in 1975, see Lange (1984). The problem of a **once-only** statistical calibration of different tracking signals and sensors was also solved for a large windfinding experiment made in 1972. The same approach was later applied to the centralised processing of Omega signal data acquired by the WMO Modular NavAid Sounding Systems of the Tropical Wind Observing Ships (TWOS) of the Global Weather Experiment (GWE) 1978-79, see Jatila (1996). True signal bearings of antipodal signals from the Omega Navigation Network were detected, see Lange (1982). Omega ceased its operation in September 1997 which caused trouble to synoptic upper-air windfinding in large areas of the Earth. Hybrid systems using both conventional tracking sensors and Navigation Aid signals (NavAids) could alleviate the situation. An important issue is how to perform the statistical fusion of different signals from many sources.

2. THE MATHEMATICAL MODELS

2.1 Conventional tracking

The physical dependencies between conventional tracking sensors and unknown Cartesian balloon co-ordinates x_b, y_b, z_b are as follows:

$$\left.\begin{aligned} h &= z_b - z_0 + 0.5\,(\rho\cos\varepsilon)^2/(R+\rho\sin\varepsilon) \\ \alpha &= \arctan\left((y_b-y_\alpha)/(x_b-x_\alpha)\right) + \delta(x_b-x_\alpha<0)\,\pi \\ \varepsilon &= \arctan\left((z_b-z_\varepsilon)/\sqrt{(x_b-x_\varepsilon)^2+(y_b-y_\varepsilon)^2}\right) \\ \rho &= \sqrt{(x_b-x_0)^2+(y_b-y_0)^2+(z_b-z_0)^2} \end{aligned}\right\} \qquad (1)$$

where h, α, ε and ρ are balloon height, azimuth, elevation and distance, respectively, which are measured, and where $\delta(x_b-x_\alpha<0) = 1$ if $x_b-x_\alpha<0$ and otherwise = 0. The known parameters are coordinates x_0, y_0, z_0 and x_α, y_α, x_ε, y_ε, z_ε of a sounding station and of its equipment, respectively. These are optical/radio theodolites or primary/secondary radars. The correction $0.5\,(\rho\cos\varepsilon)^2/(R+\rho\sin\varepsilon)$ is needed because of the Earth's curvature where R is the radius (WMO, 1996). Measurement errors have both random and systematic components. The systematic errors are caused by calibration errors of the tracking sensors.

2.2 Tracking based on terrestial NavAids

The meteorological use of NavAid signals for tracking purposes was initiated by John M. Beukers in 1964. The general physical dependency between a NavAid pseudo-distance φ (or pseudo-range, signal phase, etc.) and the three position co-ordinates x_b, y_b and z_b can be expressed locally by P. Karhunen's (1978) equation:

$$\dot{\varphi} = \dot{\rho} + \dot{x}_b \sin\lambda_\varphi + \dot{y}_b \cos\lambda_\varphi + \dot{\tau} + \dot{\gamma}_\varphi \qquad (2)$$

where λ_φ = direction of propagation of NavAid signal φ;
$\dot{\rho}$ = velocity with which the balloon is gaining distance;
$\dot{\tau}$ = phase velocity error of the local time-reference oscillator;
$\dot{\gamma}_\varphi$ = phase velocity error caused by anomalous signal propagation.

Equation (2) covers both the elliptic solution of D. T. Acheson (1974) and the standard hyperbolic solution, see R. M. Passi (1978) or Lange (1975). The four unknown quantities to be solved here are the three velocity components $\dot{\rho}$, $\dot{x}_b$, $\dot{y}_b$ and the time-reference oscillator error $\dot{\tau}$. The error $\dot{\gamma}_\varphi$ results from signal propagation anomalies and it can usually be dealt with differential NavAid correction. Thus, three NavAid signals are usually needed for upper-air windfinding as the heights of a balloon are obtained from hydrostatic computation based on the pressure, temperature and humidity (PTU) measurements. NavAid-sondes using signals from Loran-C chains, the Russian Alpha (Sigma) navigation network and VLF time-dissemination/communication transmitters have been available.

2.3 Tracking based on Loran-C signals

Strong upper winds often move the balloon to be tracked far away from the sounding station and the bearing λ_φ of a signal φ may need adjusting. Its effect is similar to the correction that is needed for balloon heights *h* due to the Earth's curvature. Phase velocity error $\dot{\tau}$ is caused by errors in tuning the local time-reference oscillator to the 100 kHz nominal frequency of Loran-C signals. Thus, the phase shift τ of an affordable crystal oscillator typically exhibits a linear trend plus random walk. The phase shift γ_φ is nearly constant in time but it may vary due to skywave reflection or other Loran-C signal anomalies (Lange, 1985).

The differential equation (2) of the observed pseudo-distances of a Loran-C signal for a balloon path can be expressed in vector form:

$$\Delta \boldsymbol{j} = \Delta \boldsymbol{\rho} + \Delta \mathbf{x}_b \sin \lambda_\varphi + \Delta \mathbf{y}_b \cos\lambda_\varphi + \Delta \boldsymbol{\tau} + \Delta \boldsymbol{\gamma}_\varphi + \Delta \mathbf{e}_\varphi \tag{3}$$

where $\Delta = \begin{bmatrix} -1 & 1 & & & \\ & -1 & 1 & & \\ & & \ddots & \ddots & \\ & & & -1 & 1 \end{bmatrix}$;

$\boldsymbol{j}$ = the vector of the observed pseudo-distances;

$\boldsymbol{\rho}$, $\mathbf{x}_b$, $\mathbf{y}_b$, $\boldsymbol{\tau}$, $\boldsymbol{\gamma}_\varphi$ = the vectors of the unknown distances, coordinates, oscillator random walk and signal anomalies, respectively; and,

$\mathbf{e}_\varphi$ = the vector of measurement errors.

The increments in equation (3) are now integrated over the balloon path by multiplying it from its left hand side by the triangular summation matrix:

$$\Sigma = \begin{bmatrix} 1 & & & \\ 1 & 1 & & \\ \vdots & & \ddots & \\ 1 & 1 & \cdots & 1 \end{bmatrix} \tag{4}$$

The consecutive positions are indexed with k, k = 1, 2, …, K. Vector $\boldsymbol{j}_1 I$ is added to both sides of (3) after the integration made by (4). It can be noted that $\boldsymbol{j}_1 \equiv \rho_1 + (x_1 - x_0)\sin\lambda_\varphi + (y_1 - y_0)\cos\lambda_\varphi + \tau_1 + c_\varphi + e_{\varphi,1}$ where c_φ is a constant $\cong \gamma_\varphi + x_0 \sin\lambda_\varphi + y_0 \cos\lambda_\varphi$ if λ_φ is about constant. Thus, the regression equation system for one Loran-C signal obtains the following vector form:

$$\boldsymbol{j} = \boldsymbol{\rho} + \mathbf{x}_b \sin\lambda_\varphi + \mathbf{y}_b \cos\lambda_\varphi + \boldsymbol{\tau} + \gamma_\varphi I + \mathbf{e}_\varphi \tag{5}$$

wherein there are four unknown vectors $\mathbf{x}_b$, $\mathbf{y}_b$, $\mathbf{z}_b$ and $\boldsymbol{\tau}$ which are the three position coordinate vectors and the time drift vector, respectively, of K consecutive positions of the balloon path. As the parameter γ_φ (or c_φ) is also unknown for each of the, say 10, Loran-C signals, there are in total 4K+10 regression parameters to be estimated. The measurement errors $\mathbf{e}_\varphi$ are non-correlated white noise, as the sampling rate is 10 per second, and the pseudo-distances $\boldsymbol{j}$ are smoothed over 5 (or 10) second non-over-lapping intervals. K is about 1024 (or 512, respectively) as the typical flight time of a balloon is less than 5120 seconds. Thus, the maximum number of measurements is 11K in case the radiosonde heights *h* are also available.

2.4 Geometrical considerations

Each NavAid signal can be used to measure just the component of a balloon motion that is parallel with its propagation direction λ_φ. Thus, NavAid-based tracking would suffer seriously from Geometric Dilution of Precision (GDOP) if all signals were nearly parallel. This is the reason why signals from distant Loran-C chains are not broadly used for windfinding. Fortunately, the regression equations (5) of all Loran-C data that are based on the physical dependency equation (2) can be augmented with regression equations of height, angular and slant-range data based on equations (1). A large non-linear equation system is obtained for each individual sounding, see the Appendix: “Regression equations for hybrid tracking systems”.

However, the NavAid tracking method does not use the accurate timing based on the pulse envelopes of Loran-C signals (Lange, 1985). This would be too expensive to be applied to consumable radio-sondes. Thus, the position of a balloon is not found here by radial geometry i.e. by simply intersecting different spheres of known location and radius. The balloon positions are estimated here by exploiting just phase data of the Loran-C signals. The positional information is therefore relative to the launch position of a balloon. Conventional tracking methods provide the positions of a balloon in a more absolute sense. These two different sources of positional information may often have small-scale conflicts that degrade windfinding severely. The different measurements must be assimilated effectively using a real-time calibration method before wind computation.

2.5 Optimum once-only calibration

The regression equation system (ii) of a hybrid tracking system is described in the Appendix, page 30. This measurement equation system contains many calibration parameters whose values must be obtained in one way or another. No practical means for physical calibration of some of these parameters exist. It is possible though to estimate all these parameters for some overdetermined equipment configurations by the statistical method of Minimum Least Squares (MLS). However, this depends on a good GDOP and synergy of the measurements. If this is not the case, additional calibration measurements are necessary. After the equations (ii) have been properly linearized the following Canonical Block-Angular (CBA) form of the measurement equations is obtained:

$$\begin{bmatrix} \Delta \boldsymbol{y}_1 \\ \Delta \boldsymbol{y}_2 \\ \vdots \\ \Delta \boldsymbol{y}_K \end{bmatrix} = \begin{bmatrix} X_1 & & & & G_1 \\ & X_2 & & & G_2 \\ & & \ddots & & \vdots \\ & & & X_K & G_K \end{bmatrix} \begin{bmatrix} \Delta \mathbf{b}_1 \\ \vdots \\ \Delta \mathbf{b}_K \\ \Delta \mathbf{c} \end{bmatrix} + \begin{bmatrix} \mathbf{e}_1 \\ \mathbf{e}_2 \\ \vdots \\ \mathbf{e}_K \end{bmatrix} \qquad (6)$$

where Δ = the operator providing departures from estimated values;
$\boldsymbol{y}_k$ = the vector of measurements for a balloon position k;
X_k= the Jacobian matrix at the position k, see Appendix;
G_k= the Jacobian matrix at the position k, see Appendix;
$\mathbf{b}_k$ = the vector of balloon coordinates at the position k;
$\mathbf{c}$ = the vector of calibration parameters; and,
$\mathbf{e}_k$ = the vector of measurement errors at the position k.

This linearized equation system (6) applies quite generally for optimum calibration of observing systems and it was originally proposed by Kaj Henricson in 1972. It was considered computationally intractable at the time. The problem was later solved and described by Lange (1975 and 1988a). It specifies a high-pass filter that can be used to damp down slow-varying calibration errors of overdetermined windfinding systems.

This calibration filter will now be generalized to work as a Kalman Filter (KF) that offers an optimal statistical method for passing calibration information forward from one sounding to the next (Lange, 1990b). Thus, these hybrid tracking systems do not need to be overdetermined all the time. Such calibration information is exploited by means of statistical regularization as if some ancillary “measurements” were made. Their error variances increase in time as the information content decreases.

3. STATISTICAL CALIBRATION

3.1 Calibration based on Kalman Filtering

Equations (7) - (9) below describe the underlying Markov process of a Kalman Filter (KF). The linearized Measurement Equation (7) tells how a measurement vector $\mathbf{y}_t$ of n components depends on a state vector $\mathbf{s}_t$ of m components and an error vector $\mathbf{e}_t$ of n components at time t:

$$\mathbf{y}_t = H_t\, \mathbf{s}_t + \mathbf{e}_t\,, \quad \text{for } t = 1, 2, \ldots \qquad (7)$$

where H_t is the Jacobian matrix that stems from the partial derivatives of the underlying physical dependencies. Here in the present balloon tracking application, each state vector $\mathbf{s}_t$ is composed of K consecutive positional vectors $\Delta\mathbf{b}_{t,1}$, $\Delta\mathbf{b}_{t,2}$, …, $\Delta\mathbf{b}_{t,K}$ (i.e. the departures from the 4-dimensional vectors of an iterated balloon trajectory where time-oscillator drifts τ_k constitute their fourth components) and of calibration parameter vector $\Delta\mathbf{c}_t$. The partitioned measurement equation (7) looks as follows:

$$\begin{bmatrix} \Delta\boldsymbol{y}_{t,1} \\ \Delta\boldsymbol{y}_{t,2} \\ \vdots \\ \Delta\boldsymbol{y}_{t,K} \end{bmatrix} = \begin{bmatrix} X_{t,1} & & & & G_{t,1} \\ & X_{t,2} & & & G_{t,2} \\ & & \ddots & & \vdots \\ & & & X_{t,K} & G_{t,K} \end{bmatrix} \begin{bmatrix} \Delta\mathbf{b}_{\mathbf{t,1}} \\ \vdots \\ \Delta\mathbf{b}_{\mathbf{t,K}} \\ \Delta\mathbf{c}_{\mathbf{t}} \end{bmatrix} + \begin{bmatrix} \mathbf{e}_{\mathbf{t,1}} \\ \mathbf{e}_{\mathbf{t,2}} \\ \vdots \\ \mathbf{e}_{\mathbf{t,K}} \end{bmatrix} \qquad (8)$$

where $\mathbf{y}_t = [\Delta\boldsymbol{y}_{t,1}', \Delta\boldsymbol{y}_{t,2}', \ldots, \Delta\boldsymbol{y}_{t,K}']'$ and its element $\Delta\boldsymbol{y}_{t,k}$ is the vector of observed departures from the expected values of measurements that have been computed from the physical dependency equations (1) - (2) and where the element $\mathbf{e}_{t,k}$ is the vector representing respective measurement errors for k = 1, 2,…,K. Thus, $X_{t,1}$, $X_{t,2}$,…, $X_{t,K}$ are the Jacobian matrices of partial derivatives of the local physical dependencies and $G_{t,1}$, $G_{t,2}$,…, $G_{t,K}$ are the Jacobian matrices describing how the measurements are locally affected by errors in the calibration parameters represented here by $\Delta\mathbf{c}_\mathbf{t}$.

Equation (9) describes time evolution of an overall system at time t. It is the linearized System (or State) Equation:

$$\mathbf{s}_t = A_t\, \mathbf{s}_{t-1} + B_t\, \mathbf{u}_{t-1} + \mathbf{a}_t \quad \text{for } t = 1, 2, \ldots \text{ (and, } \mathbf{s}_0 \text{ given)} \qquad (9)$$

where matrix A_t is the state transition (Jacobian) matrix and B_t is the control/calibration gain (Jacobian) matrix. The equation tells how present state $\mathbf{s}_t$ of the overall system develops from its previous state $\mathbf{s}_{t-1}$ when it is also affected by control/calibration forcings $\mathbf{u}_{t-1}$ and random errors $\mathbf{a}_t$. Calibration adjustments and drifts are contained in vectors $\mathbf{u}_{t-1}$ and $\mathbf{a}_t$, respectively.

Here in the balloon tracking experiment, matrices A_t and B_t are primarily composed of zero elements, as very simple random walk models apply both to the consecutive balloon positions of a sounding, and to the calibration parameters of tracking equipment from a previous sounding t-1 to the next sounding t.

When measurement errors $\mathbf{e}_t$ and system noises $\mathbf{a}_t$ are neither auto- nor cross-correlated, they do not correlate with $\mathbf{s}_0$ and the covariances are:

$$\left.\begin{array}{l} R_t = \mathrm{Cov}(\mathbf{e}_t) = E(\mathbf{e}_t\, \mathbf{e}_t') \\ Q_t = \mathrm{Cov}(\mathbf{a}_t) = E(\mathbf{a}_t\, \mathbf{a}_t') \end{array}\right\} \qquad (10)$$

then the *Kalman forward recursions* from equations (11) - (14) give the Best Linear Unbiased Estimates (BLUE) $\hat{\mathbf{s}}_t$ of present states $\mathbf{s}_t$ as follows:

$$\hat{\mathbf{s}}_t = \tilde{\mathbf{s}}_t + K_t\, (\mathbf{y}_t - H_t\, \tilde{\mathbf{s}}_t) \qquad (11)$$

where $\tilde{\mathbf{s}}_t$ is predicted using either an underlying physical model or the State Equation (9) as follows:

$$\tilde{\mathbf{s}}_t = A_t\, \hat{\mathbf{s}}_{t-1} + B_t\, \mathbf{u}_{t-1} \qquad (12)$$

and where the error covariance matrices for the prediction (simulation) and the BLUE estimation, respectively, are given as follows:

$$\left.\begin{array}{l} \tilde{P}_t = \mathrm{Cov}(\tilde{\mathbf{s}}_t - \mathbf{s}_t) = E\{(\tilde{\mathbf{s}}_t - \mathbf{s}_t)(\tilde{\mathbf{s}}_t - \mathbf{s}_t)'\} = A_t\, \hat{P}_{t-1}\, A_t' + Q_t \\ \hat{P}_t = \mathrm{Cov}(\hat{\mathbf{s}}_t - \mathbf{s}_t) = E\{(\hat{\mathbf{s}}_t - \mathbf{s}_t)(\hat{\mathbf{s}}_t - \mathbf{s}_t)'\} = \tilde{P}_t - K_t\, H_t\, \tilde{P}_t \end{array}\right\} \qquad (13)$$

and where the *Kalman gain matrix* K_t for t = 1, 2,… is computed from:

$$K_t = \tilde{P}_t\, H_t'\, (H_t\, \tilde{P}_t\, H_t' + R_t)^{-1} \qquad (14)$$

These recursions (11) - (14) are usually initialized by setting $\hat{\mathbf{s}}_0 = E\mathbf{s}_0$ $(\cong \vec{0})$ and $\hat{P}_0 = \mathrm{Cov}(\hat{\mathbf{s}}_0 - \mathbf{s}_0) = \mathrm{Cov}(\mathbf{s}_0)$, which are assumed to be known (see e.g. Jazwinski, 1970).

The recursive linear solution of $\hat{\mathbf{s}}_t$, given above by equations (11) - (14), optimally updates the mean value vector $\hat{\mathbf{s}}_{t-1}$. The computational solution requires the matrix $H_t \tilde{P}_t H_t' + R_t$ of size $n \times n$ to be inverted where n is the number of measurements in vector $\mathbf{y}_t$. These recursion formulas are good for Kalman Filtering applications where a small number of measurements are available during each small time interval] t-1,t]. It is the fastest way of **once-only** updating a mean value vector. However, there is a serious risk of erroneous feedback that may accumulate in repeated corrections.

Continued maintenance of calibration parameters by statistical means is more complex than the case of a once-only calibration. All involved calibration parameters must be continuously *observable*. The concept of *observability* means that all state parameters are uniquely determinable by statistical means (Grewal et al., 1993). This is possible only if there is a sufficient degree of overdetermination in the combined system of linear equations (7) and (9) for a balloon flight at time t (t = 1, 2,...). As all measurements depend more or less linearly on their calibration parameters, there must be a lot of data redundancy to make it possible to detect a calibration error. The flight time of a balloon should be as long as possible because the total number $n+m$ of the combined equations must be significantly larger than the number m of all the state parameters to be estimated. The calibration parameters undergo random walk and the synergy of different signals/sensors may also vary from sounding to sounding.

These calibration problems have been analysed only late-hand in large intercomparison experiments exploiting simultaneous measurements from many different tracking instruments (Lange, 1984). Fortunately, the real-time statistical calibration based on optimal Kalman Filtering is also reliable (i.e. stable) under the *stability conditions* to be outlined next.

3.2 Stability of repeated statistical calibration

It is necessary and sufficient for an optimal linear Kalman Filter to be stable that its observability and controllability conditions are satisfied (Kalman, 1960). Non-linear systems are usually handled using local linear approximations, e.g. by Extended Kalman Filtering (EKF). However, the observability and controllability conditions are still necessary but not absolutely sufficient to guarantee the stability of Kalman Filtering. We briefly outline these conditions in the following:

Optimality of a Discrete Kalman Filter (DKF) means that all equations (7) - (14) are absolutely valid. This is, of course, never true in the real world. Thus, all practical implementations of Kalman Filtering are more or less suboptimal. If a selected Kalman Filter solution is made as near to optimal as possible then all real-world effects must be taken into account. True optimality can be achieved only by meticulous physical modelling and estimating all uncertainties properly;

Observability means that the Kalman Filter is capable of estimating all *m* components of the state parameter vector $\mathbf{s}_t$, by extracting the information that is contained in the measurement vectors $\mathbf{y}_t$, $\mathbf{y}_{t-1}$,..., $\mathbf{y}_1$ and $\mathbf{s}_0$. If the observability condition is not satisfied then uncertainty in the estimated values grows beyond acceptable tolerances according to equations (9) and (13). Many calibration parameters are not observable at all if there is no prior information on them or if the measurements depend on them in the same linear way as other state parameters. Equations (1) are sufficiently non-linear for estimating the calibration parameters only if long time series of different measurements are analysed simultaneously;

Controllability is an important property of a system to be controlled. A controllable system may be defined as a system which can be steered to any state $\mathbf{s}_t$ from initial state $\mathbf{s}_0$ by using some controls $\mathbf{u}_0$, $\mathbf{u}_1$,…, $\mathbf{u}_{t-1}$ that are available. Controllability of an observing system is also important. If the sensors undergo physical calibration then:

1) the numerical values of calibration must be given in $\mathbf{u}_0$, $\mathbf{u}_1$,…, $\mathbf{u}_{t-1}$; and
2) there must be sufficient gain in matrix B_t of equation (9) upon the calibration parameters in $\mathbf{s}_t$ in the sense that their random walk specified in system noise vector $\mathbf{a}_t$ does not become superfluous.

Stability of an optimal linear Kalman Filter can be monitored by computing its error covariances $\tilde{P}_t$ and $\hat{P}_t$ from equations (13). The filter is unstable when these error covariances obtain excessively large values. This happens if system noises $\mathbf{a}_t$, $\mathbf{a}_{t-1}$,…, $\mathbf{a}_1$ overrun the combined signal from observational data $\mathbf{y}_t$, $\mathbf{y}_{t-1}$,…, $\mathbf{y}_1$ and $\hat{\mathbf{s}}_0$. The stability of an observing system is controlled either by making more or better observations for equation (7) or by improving physical modelling for equation (9).

There is also a difficult computational problem. The matrix to be inverted in equation (14) for our balloon tracking experiment is of the order of 11Kx11K (K=1024). This is such a demanding High Performance Computing (HPC) task that without using the FKF method the statistical calibration based on Kalman Filtering would be too difficult to apply operationally at ordinary upper-air sounding stations.

4. THE FKF METHOD

4.1 Harvey's approach

The System Equation (9) is written into the following equivalent form:

$$A_t\ \hat{\mathbf{s}}_{t-1} + B_t\ \mathbf{u}_{t-1} = I\ \mathbf{s}_t + A_t\ (\hat{\mathbf{s}}_{t-1} - \mathbf{s}_{t-1}) - \mathbf{a}_t \qquad (15)$$

and combined with Measurement Equation (7) in order to obtain the so-called Augmented Model:

$$\begin{bmatrix} \mathbf{y_t} \\ \mathbf{A_t}\hat{\mathbf{s}}_{\mathbf{t-1}} + \mathbf{B_t}\mathbf{u}_{\mathbf{t-1}} \end{bmatrix} = \begin{bmatrix} H_t \\ I \end{bmatrix} \mathbf{s}_t + \begin{bmatrix} \mathbf{e_t} \\ \mathbf{A_t}(\hat{\mathbf{s}}_{\mathbf{t-1}} - \mathbf{s}_{\mathbf{t-1}}) - \mathbf{a_t} \end{bmatrix} \qquad (16)$$

i.e. $\mathbf{z}_t = Z_t\ \mathbf{s}_t + \eta_t$

The state parameters may often be estimated by attempts of solving the following Regression Analysis problem:

$$\hat{\mathbf{s}}_t = (Z_t'\ V_t^{-1}\ Z_t)^{-1}\ Z_t'\ V_t^{-1}\mathbf{z_t} \qquad (17)$$

and

$$\hat{P}_t = \mathrm{Cov}(\hat{\mathbf{s}}_t - \mathbf{s}_t) = (Z_t'\ V_t^{-1}\ Z_t)^{-1} \qquad (18)$$

where $V_t = \mathrm{Cov}(\eta_t) = E\ (\eta_t\ \eta_t')$.

Formulas (17) - (18) are statistically but not numerically equivalent to the Kalman recursions (11) - (14), see Harvey (1981). The size of error covariance matrix V_t is $(n+m)\mathrm{x}(n+m)$. Its inversion is not a big problem as it can always be made close to block-diagonal, see Lange (1997). However, the outer inversion in Formula (18) is of the order of 4Kx4K. This is still too large to be done on computers used for operational tracking of weather balloons. Fortunately, the matrix $Z_t'V_t^{-1}Z_t$ is sparse and has a bordered block-diagonal (or bordered band-diagonal) structure.

4.2 The FKF formulas for calibration

The Regression Analysis model (16) of a hybrid tracking system shrinks as A_t = diag(0, I), $B_t = [0]$ and $G_{t,0} = I$, after discarding all non-informative equations, to the following model:

$$\begin{bmatrix} \mathbf{y}_t \\ \Delta\hat{\mathbf{c}}_{t-1} \end{bmatrix} = \begin{bmatrix} H_t & \\ 0 & I \end{bmatrix} \mathbf{s}_t + \begin{bmatrix} \mathbf{e}_t \\ (\Delta\hat{\mathbf{c}}_{t-1} - \Delta\mathbf{c}_{t-1}) - \mathbf{a}_{t,c} \end{bmatrix} \qquad (19)$$

where $\mathbf{y}_t = [\Delta\boldsymbol{y}_{t,1}', \Delta\boldsymbol{y}_{t,2}',\ldots, \Delta\boldsymbol{y}_{t,K}']'$, $\mathbf{s}_t = [\Delta\mathbf{b}_{t,1}', \Delta\mathbf{b}_{t,2}',\ldots, \Delta\mathbf{b}_{t,K}', \Delta\mathbf{c}_t']'$, $\mathbf{e}_t = [\mathbf{e}_{t,1}', \mathbf{e}_{t,2}',\ldots, \mathbf{e}_{t,K}']'$ and $\mathbf{a}_t = [\mathbf{a}_{t,1}', \mathbf{a}_{t,2}',\ldots, \mathbf{a}_{t,K}', \mathbf{a}_{t,c}']'$. The FKF formulas for updating the calibration parameter estimates $\Delta\hat{\mathbf{c}}_t$ and solving the positional estimates $\Delta\hat{\mathbf{b}}_{t,1}$, $\Delta\hat{\mathbf{b}}_{t,2},\ldots, \Delta\hat{\mathbf{b}}_{t,K}$ at sounding times t = 1, 2,… are then as follows:

$$\left.\begin{aligned} \Delta\hat{\mathbf{b}}_{t,k} &= (X_{t,k}' V_{t,k}^{-1} X_{t,k})^{-1} X_{t,k}' V_{t,k}^{-1} (\Delta\boldsymbol{y}_{t,k} - G_{t,k}\, \Delta\hat{\mathbf{c}}_t) \\ &\qquad\qquad \text{for } k = 1, 2,\ldots, K \\ \Delta\hat{\mathbf{c}}_t &= \{ \sum_{k=0}^{K} G_{t,k}' R_{t,k} G_{t,k} \}^{-1} \sum_{k=0}^{K} G_{t,k}' R_{t,k} \Delta\boldsymbol{y}_{t,k} \end{aligned}\right\} \qquad (20)$$

where for k = 0:

$R_{t,0} = [\mathrm{Cov}(\Delta\hat{\mathbf{c}}_{t-1} - \Delta\mathbf{c}_{t-1}) + \mathrm{Cov}(\mathbf{a}_{t,c})]^{-1}$

$\Delta\boldsymbol{y}_{t,0} = \Delta\hat{\mathbf{c}}_{t-1}$

$R_{1,0} = [\mathrm{Cov}(\mathbf{c}_0) + \mathrm{Cov}(\mathbf{a}_{1,c})]^{-1}$

$\mathbf{c}_0$ = vector of initial calibration

$\Delta\boldsymbol{y}_{1,0} = 0$

and, for k = 1, 2,…, K:

$R_{t,k} = V_{t,k}^{-1} - V_{t,k}^{-1} X_{t,k} (X_{t,k}' V_{t,k}^{-1} X_{t,k})^{-1} X_{t,k}' V_{t,k}^{-1}$

$V_{t,k} = \mathrm{Cov}(\mathbf{e}_{t,k})$.

Let it be emphasized that these FKF formulas (20) are really fast to compute. Matrix inversions of only size 4x4 and of about 20x20 are needed for estimating the calibration parameters of a large hybrid tracking system. These simple FKF formulas can be derived analytically from equation (17) only by knowing Formula (21) when the matrix $Z_t'V_t^{-1}Z_t$ is bordered block-diagonal. The matrix V_t is not tedious to invert as it is typically block-diagonal. If it is not block-diagonal then it must be made such by introducing new calibration parameters that explain and remove the unwanted error covariances from the model (19), see Lange (1997).

Error variances and covariances of all the estimated parameters can be obtained by computing the following covariance matrix:

$$\hat{P}_t = \mathrm{Cov}(\hat{\mathbf{s}}_t - \mathbf{s}_t) = (Z_t'\, V_t^{-1}\, Z_t)^{-1}$$

$$= \begin{bmatrix} C_1 + D_1SD_1' & D_1SD_2' & \cdots & D_1SD_K' & -D_1S \\ D_2SD_1' & C_2 + D_2SD_2' & & D_2SD_K' & -D_2S \\ \vdots & & \ddots & \vdots & \vdots \\ D_KSD_1' & D_KSD_2' & \cdots & C_K + D_KSD_K' & -D_KS \\ -SD_1' & -SD_2' & \cdots & -SD_K' & S \end{bmatrix} \qquad (21)$$

where $S = \{ \sum_{k=0}^{K} G_{t,k}'\mathit{R}_{t,k}G_{t,k}\}^{-1}$

and, for k= 1, 2, …, K:

$C_k = (X_{t,k}'\, V_{t,k}^{-1}\, X_{t,k})^{-1}$

$D_k = (X_{t,k}'\, V_{t,k}^{-1}\, X_{t,k})^{-1}\, X_{t,k}'\, V_{t,k}^{-1}G_{t,k}$.

The error covariance matrix $\hat{P}_t$ above is immense but there is no absolute need to compute it. Its blocks S and $C_k+D_kSD'_k$ (k = 1, 2,…, K) can be computed one at a time and they are needed for monitoring the stability of an optimal FKF filtering process.

Formulas (20) - (21) can be easily derived by making use of the bordered block-diagonal structure of the matrix $Z_t'V_t^{-1}Z_t$ to be inverted and of Frobenius' formula:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} = \begin{bmatrix} A^{-1} + A^{-1}BH^{-1}CA^{-1} & -A^{-1}BH^{-1} \\ -H^{-1}CA^{-1} & H^{-1} \end{bmatrix} \text{ where } H = D - CA^{-1}B \qquad (22)$$

4.3 Calibration of hybrid tracking systems

Operational windfinding systems can be equipped, temporarily at least, with redundant sensors for producing measurement data $\mathbf{y}_t$ (t = 1, 2, …) with some overdetermination. Performance of the system can then be checked by monitoring internal consistency of the measurements (Lange, 1983). Required corrective actions e.g. built-in-calibration can then be made on call by Kalman Filtering.

The Kalman Filtering can be based on Weighted Least Squares (WLS) computations when the residual errors of measurements are first orthogonalized. The complexity of the involving inversion problem usually calls for the Fast Kalman Filtering (FKF) method in Formulas (20) to be used for any large hybrid system. Formula (21) indicates when an operational tracking system becomes unstable. Modern sensors yield measurements at high sampling rates which help to keep the observability condition of an optimal or near-optimal linear Kalman Filter satisfied. The Covariance Intersection (C/I) method of J. K. Uhlmann (1995) outlines a procedure of modifying the measurement/system error covariances of equations (10) for a conservative estimation when these covariances are more or less unknown.

The Hybrid Windfinding Algorithm (HWA) package used in this study monitors stability of the FKF calculations by computing the error variances given in the diagonal of $\hat{P}_t$ (t = 1, 2,…) of Formula (21). If and when these error variances grow beyond some acceptable tolerances then overdetermination must be increased in one way or another.

5. STATISTICAL ACCURACY ESTIMATION

5.1 Estimation of error variances

Susan D. Horn’s, Roger A. Horn’s and David B. Duncan’s (1975) Almost Unbiased Estimation (AUE) method based on C. R. Rao’s (1972 and 1975) Minimum Norm Quadratic Unbiased Estimation (MINQUE) method has been applied. These two estimation methods allow quite generally that the error covariance matrices $R_t = Cov(\mathbf{e}_t) = E(\mathbf{e}_t\mathbf{e}_t')$ of measurements $\mathbf{y}_t$ (t = 1, 2,…) need not be of full rank nor diagonal. Fortunately, the case of a typical balloon tracking system is much simpler. The vector of error variances to be estimated by Rao’s MINQUE (with an assumption of the so-called translation invariance) takes the following simple form:

$$\hat{\sigma} = [\hat{\sigma}_1^2,\ \hat{\sigma}_2^2, ...,\ \hat{\sigma}_N^2]' = F^{-1}\,\mathbf{q} \qquad (23)$$

where N = $n+m$ and $\mathbf{q}$ is the vector of squared values of components of the computed residual vector η_t of an Augmented Model (16) and F is a square matrix.

The residual vector η_t, i.e. an *innovation sequence* of a Kalman Filter, can be estimated by the respective FKF computations. However, the matrix F and its inverse matrix F^{-1} are quite tedious to compute. Furthermore, the computed error variances may sometimes obtain negative values due to the unbiasedness requirement of the quadratic estimators. Fortunately, good approximations of these error variances can also be obtained by using the AUE method. These estimates are always non-negative and much easier to compute.

The variance component estimates given by equation (23) depend to some degree on their a priori values that have been used in the FKF computations. Thus, iterative estimation procedures have been proposed and studied e.g. by G. B. Goldstein (1969). Such methods for estimating true absolute magnitudes of these error variance components are known as "bootstrapping" but are not applied here by the HWA software.

5.2. Winds and windfinding accuracies

The cubic smoothing spline routine of Carl de Boor (1978) is used for estimating winds and their accuracies from statistically calibrated balloon tracks as reported in Lange (1988a). The interpolation accuracy of a piecewise cubic polynomial $x_b = \mathbf{p}' \, \mathbf{t} = [p_3, p_2, p_1, p_0][t^3, t^2, t^1, 1.0]'$ can be computed from:

$$\mathrm{Var}(x_b) = \mathbf{t}' \, \mathrm{Cov}(\mathbf{p}) \, \mathbf{t} \qquad (24)$$

Coefficient vector **p** and its covariance matrix Cov(**p**) are provided by the smoothing cubic spline computation. Thus, the error variance of an estimated wind component $\dot{x}_b = \mathbf{p}' \, \dot{\mathbf{t}} = [p_3, p_2, p_1, p_0][3t^2, 2t^1, 1.0, 0.0]'$ is similarly obtained by the same time-differentation as follows:

$$\begin{aligned}\mathrm{Var}(\dot{x}_b) &= \dot{\mathbf{t}}' \, \mathrm{Cov}(\mathbf{p}) \, \dot{\mathbf{t}} \\ &= [3t^2, 2t^1, 1.0, 0.0] \; \mathrm{Cov}(\mathbf{p}) \, [3t^2, 2t^1, 1.0, 0.0]'\end{aligned} \qquad (25)$$

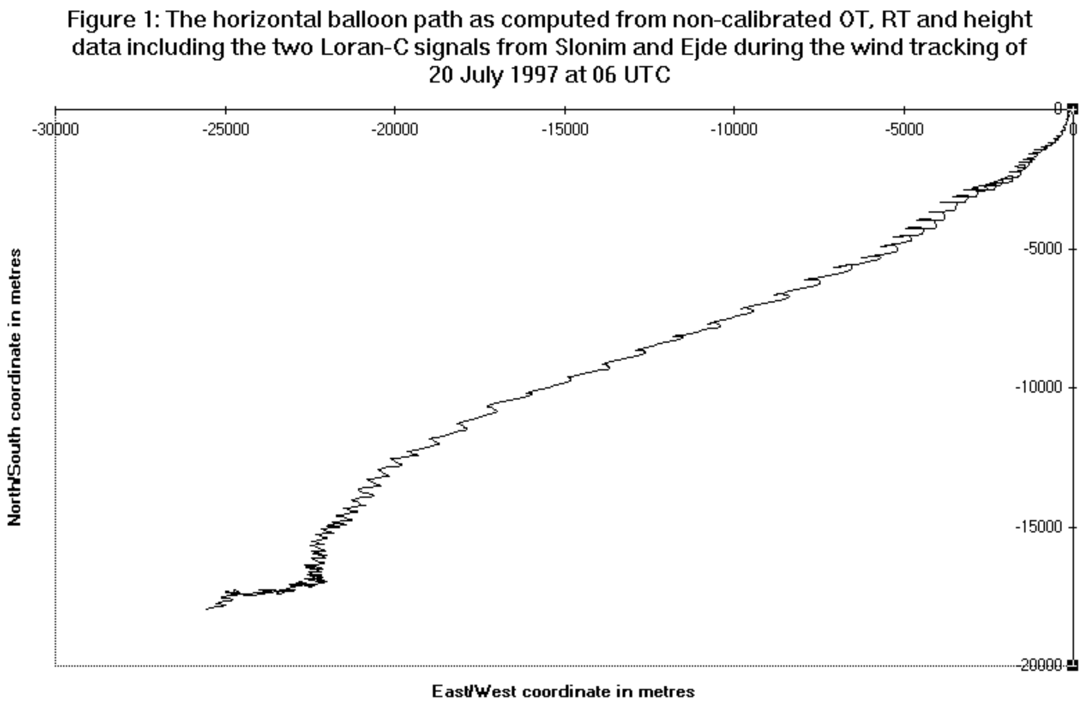
6. BALLOON TRACKING EXPERIMENT

Signals from the two Loran-C chains of Ejde (9007) and Bryansk (8000), azimuth and elevation data from an automatic ME12 radio theodolite (RT), and manual readings from an optical theodolite (OT) were collected during the balloon tracking experiment at Upper-air Sounding Station 02935 of Tikkakoski, Finland, 18-20 July 1997. A Vaisala DigiCORA II sounding system provided height observations from Vaisala RS80 radiosondes based on hydrostatic computation. The five hybrid experimental soundings made at 06 and 18 UTC were part of the daily routine observations of Station 02935. The balloon tracking for upper-air windfinding is normally based on Loran-C signals only.

During the experiment, the balloon tracking computations were made with the Hybrid Windfinding Algorithm (HWA) software. The OT and the RT data improved overdetermination for a better observability of calibration parameters. All the tracking signals/sensors were statistically calibrated by using the FKF method. The time step used for the FKF filtering was 12 hours. Each data window consisted of an entire sounding of up to 80 minutes. The Loran-C and RT data were recorded at intervals of 5 and 10 seconds, respectively. The optical readings are most accurate but were available only at 60-second intervals due to the manual recording used. The overdetermination needed for observability of all calibration parameters was mainly based on the data-redundancy of the Loran-C signals. However, the accuracy of the statistical calibration results relies essentially on the OT data.

Different combinations of the Loran-C signals and the angular measurements were experimented in order to demonstrate the importance of overdetermination for a stable statistical calibration. However, the main outcome was that statistical calibration seems to be a necessity in hybrid balloon tracking systems.

Figures 1 and 2 show the estimated maximum likelihood horizontal paths of the last balloon flight with and without the statistical calibration based on the FKF method. These hybrid computations were based on the two Loran-C signals of Slonim (8000-2) and Ejde (9007-M) as well as on the respective height, OT and RT data.



The emerged saw-tooth feature in Figure 1 is not realistic at all. The balloon drifted smoothly in North-Easterly winds as shown in Figure 2.

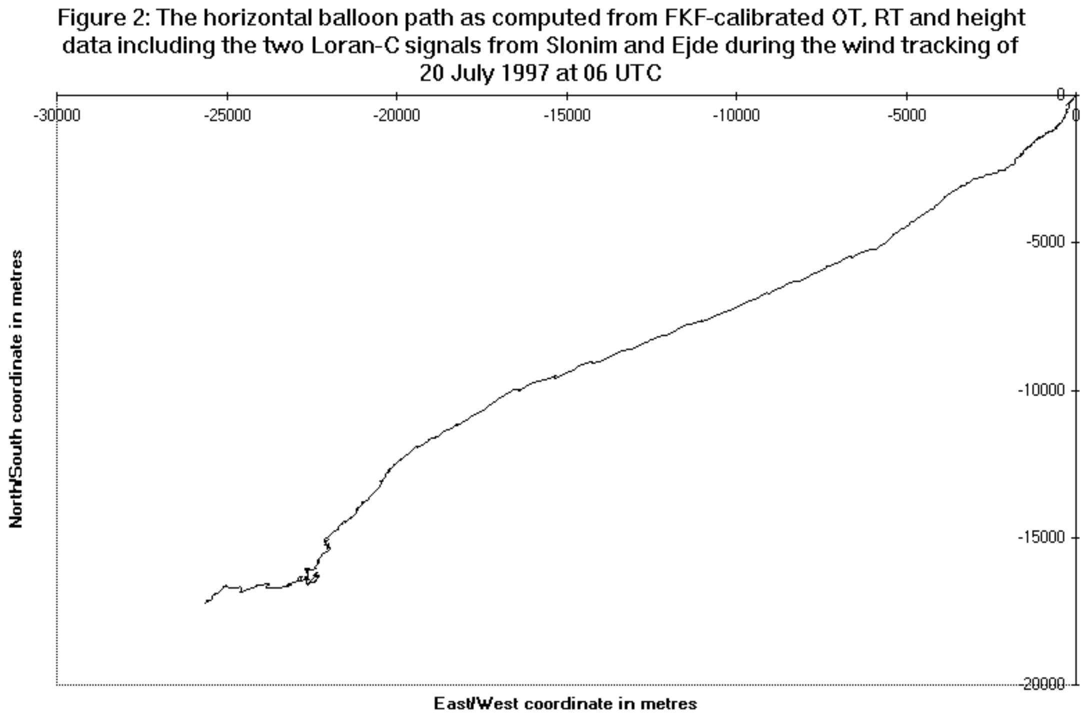


Figure 3 reveals that the saw-teeth were caused by a 4^{o} systematic error in the azimuth of RT. The 10-second and the 60-second azimuth readings from the two theodolites are superimposed on simulated values. The OT azimuth values given only at the 60-second intervals corrected the distracted estimates of the computed balloon path only intermittently.

Figure 3: The observed and the simulated OT and RT azimuth values obtained from the non-calibrated balloon tracking of 20 July 1997 at 06 UTC

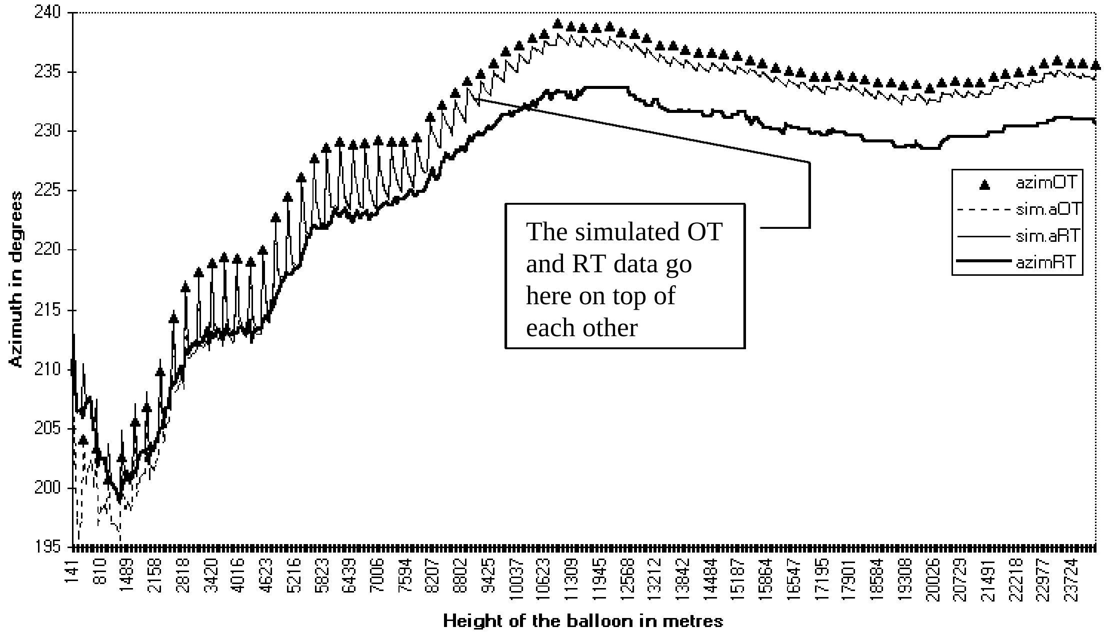
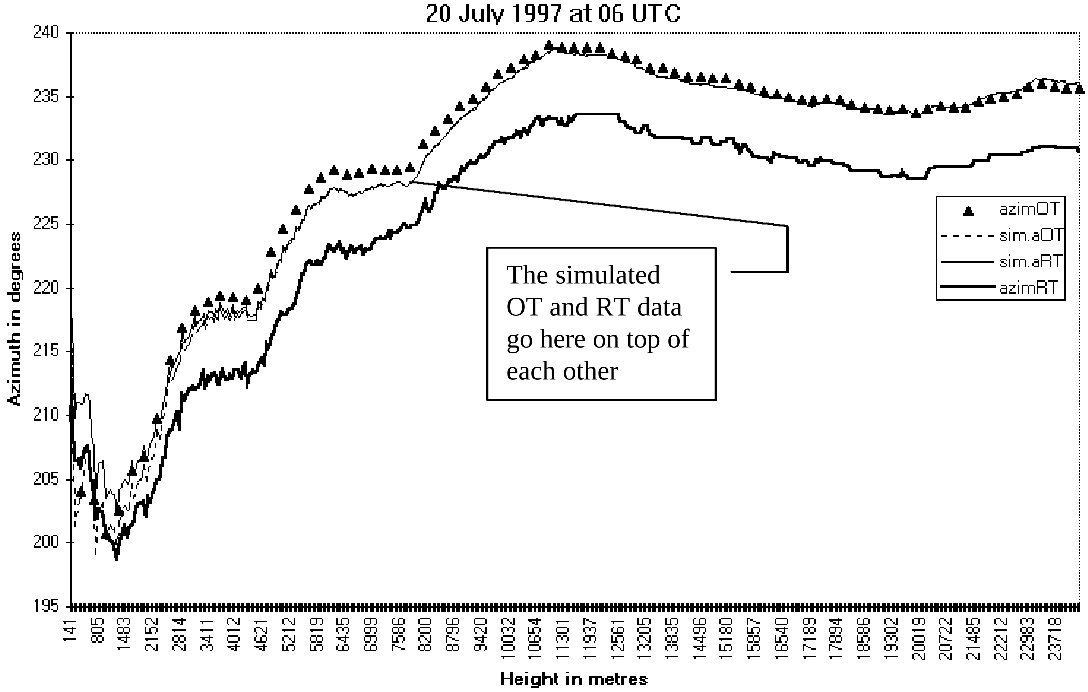


Figure 4 below exhibits disappearance of the saw-tooth problem after the statistical calibration was made by the FKF method. The RT azimuths are, however, plotted below without the 4^o correction for clarity’s sake:

Figure 4: The observed and the simulated OT and RT azimuth values obtained from the FKF-calibrated balloon tracking of 20 July 1997 at 06 UTC



Figures 5 and 6 show how the simulated Loran-C pseudo-distances from Ejde (9007-M) and Slonim (8000-2) compare differently with the observed signals before and after the statistical calibration made by FKF.

Figure 5: The observed and the simulated Loran-C pseudo-distances from Ejde and Slonim as obtained from the non-calibrated balloon tracking of 20 July 1997 at 06 UTC

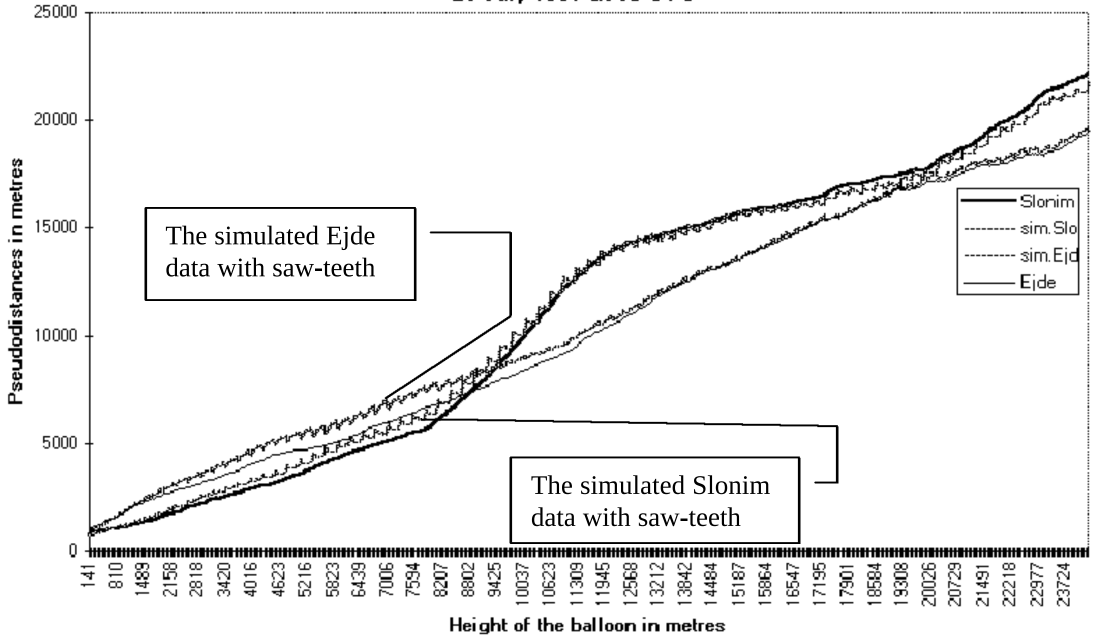


Figure 6: The observed and the simulated Loran-C pseudo-distances from Ejde and Slonim as obtained from the FKF-calibrated balloon tracking of 20 July 1997 at 06 UTC

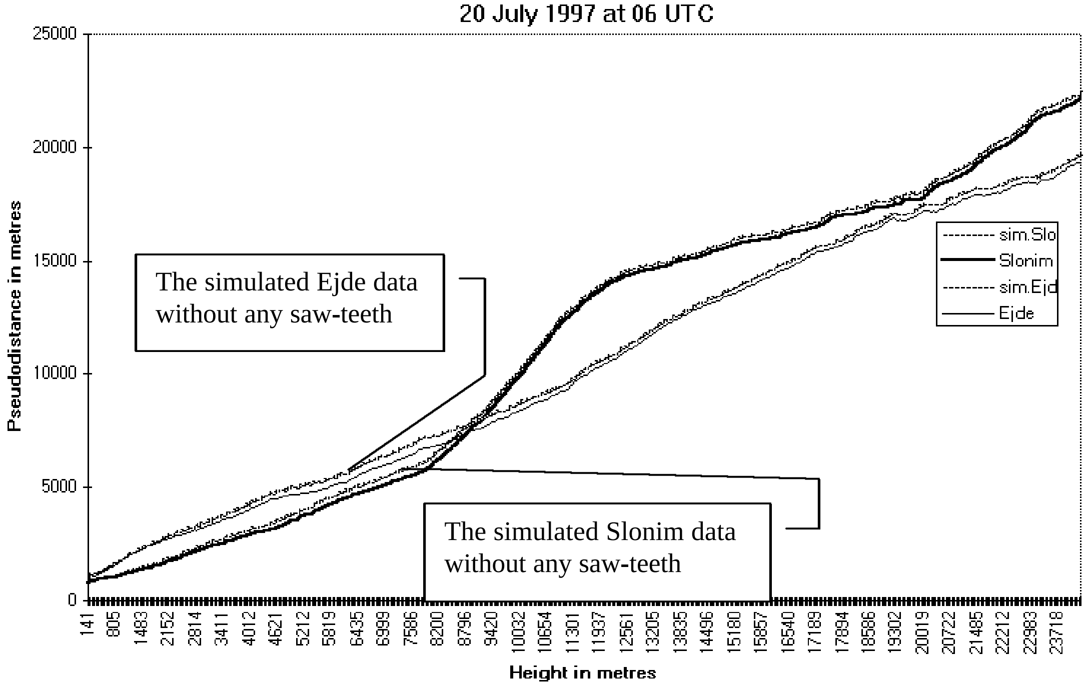
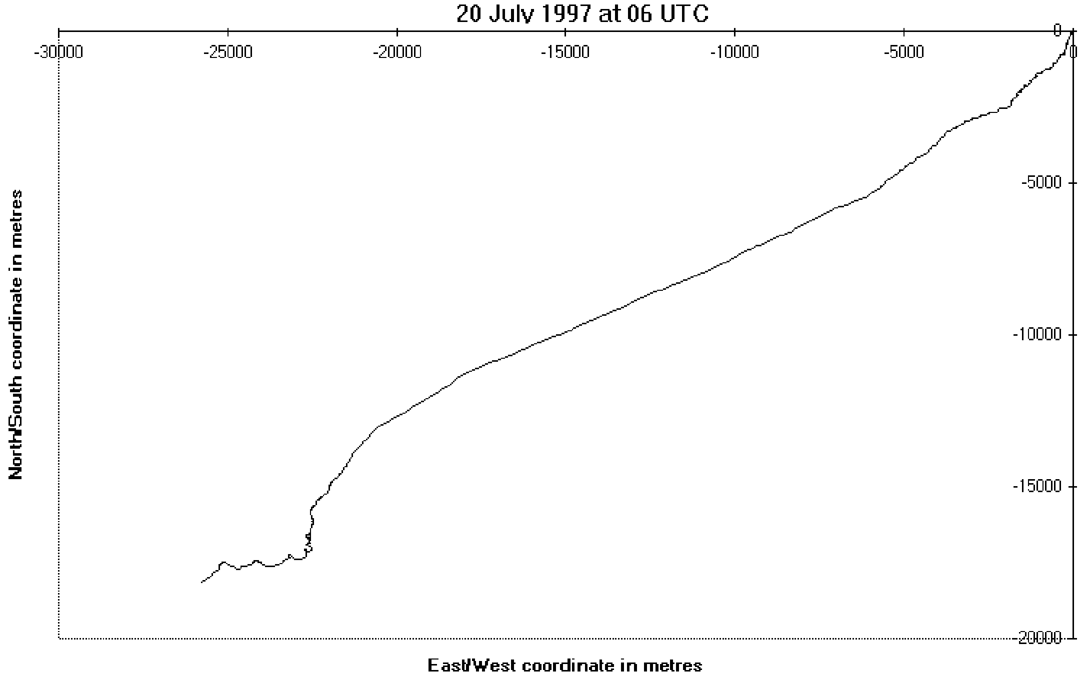


Figure 6 above confirms the proposed cause of the saw-tooth problem as the simulated Loran-C data comply remarkably well with the real data.

Figure 7 gives the maximum likelihood horizontal path obtained from a HWA computation when all available measurements were used:

Figure 7: The horizontal balloon path as computed from OT, RT and height data including all 10 available Loran-C signals from the two chains of Ejde and Bryansk during the tracking of 20 July 1997 at 06 UTC



There was no significant difference before and after the FKF calibration.

7. CONCLUDING REMARKS

Numerical experimentation was made with the HWA algorithm using different selections of tracking measurements. These indicate that statistical calibration becomes the more dependent on FKF, the fewer sensors are available. This is because the observability condition requires time series of a considerable length when data from only a few sensors are available. Observability of calibration parameters depends much on the Geometric Dilution Of Precision (GDOP) that varies from flight to flight. Well-controlled subsystems like a precision radar or a number of high-quality optical theodolites can be temporarily employed to facilitate the statistical calibration of a balloon tracking system. Modern Loran-C and GPS-based NavAid-tracking systems may also provide good reference data at high sampling rates. However, their serious limitations either in geogaphical coverage or in signal penetration suggest hybrid navigation solutions.

References

(l) Legendre, A-M. (1806): “Nouvelles Methodes pour la Determination des Orbits des Cometes.” Paris.

(2) Gauss, K. F. (1809): “Werke.” Gottingen, 4, 1821 (collected works, 1873). See also “Theory of the Motion of the Heavenly Bodies Moving about the Sun in Conic Sections.” Dover, New York.

(3) Kalman, R. E. (1960): “A New Approach to Linear Filtering and Prediction Problems.” Trans. ASME J. of Basic Eng., 82-D, pp. 35-45.

(4) Kalman, .R. E. and Bucy, R. S. (1961): “New Results in Linear Filtering and Prediction Theory.” Trans. ASME J. of Basic Eng., 83-D, pp. 95-108.

(5) Goldstein, G. B. (1969): “Estimating linear regression parameters in the presence of non-white Gaussian noise with unknown covariance parameters.” University of Southern California, Ph.D., Engineering, electrical, 114 p.

(6) Jazwinski, A. H. (1970): “Stochastic Processes and Filtering Theory.” Academic Press, New York and London.

(7) Rao, C. R. (1972): “Estimation of variance and covariance components in linear models.” Journal of the American Statistical Association, March 1972, Volume 67, Number 337, pp. 112-115.

(8) Acheson, D. T. (1974): “Omega windfinding and GATE.” Bulletin of the American Meteorological Society, Volume 55, No. 33, May 1974, pp. 385-398.

(9) Gelb, A. editor (1974): “Applied Optimal Estimation.” The MIT Press, Cambridge, Mass.

(10) Lange, A. (1975): “TULU”-projektin loppuraportti. Kriisiajan sääpalvelun kehittämistoimikunta. MATINE, Helsinki, 1975.

(11) Rao, C. R. (1975): “Linear Statistical Inference and Its Applications – Second Edition.” John Wiley & Sons, Inc., New York – London – Sydney.

(12) Horn, S. D., Horn, R. A. and Duncan, D. B. (1975): “Estimating hetero-scedastic variances in linear models.” Journal of the American Statistical Association, June 1975, Volume 70, Number 350, pp. 380-385.

(13) Karhunen, P. (1978): “Yleinen Omega-verkkoon perustuva tuulenmittausmenetelmä.” Lic. Tech. (PhD) thesis, Helsinki University of Technology, Helsinki, Finland.

(14) de Boor, C. (1978): “A Practical Guide to Splines.” Springer, New York.

(15) Passi, R. M. (1978-79): “Overdetermined Windfinding Systems.” Systems and Techniques for Synoptic Windfinding, Atmospheric Technology, Number 10 - Winter 1978-79, National Center for Atmospheric Research, Boulder, Colorado, pp. 65-75.

(16) Harvey, P. (1981): “Time Series Models.” Philip Allan Publishers Ltd, Oxford, UK, see pages 101-119.

(17) Lange, A. (1982): “Multipath propagation of VLF Omega signals.” IEEE PLANS '82 - Position Location and Navigation Symposium Record, December 1982, pp. 302-309.

(18) Lange A. (1983): “Detection and remedy of signal propagation problems with the VLF Omega transmission used in tracking applications.” Fifth Symposium Meteorological Observations and

Instrumentation Preprints, Toronto, Ont., Canada, April 11-15, 1983. American Meteorological Society, USA, pp. 116-122.

(19) Lange, A. (1984): “Integration, calibration and intercomparison of windfinding devices.” WMO Instruments and Observing Methods Report No. 15, World Meteorological Organization, Geneva, Switzerland, 1984, pp. 41-45.

(20) Lange, A. (1985): “Meteorological observations using NavAid methods.” Technical Note No. 185, WMO – No. 641, World Meteorological Organization, Geneva, Switzerland, 45 pages.

(21) Lange, A. (1988a): “A High-pass Filter for Optimum Calibration of Observing Systems with Applications.” Simulation and Optimization of Large Systems, edited by A. J. Osiadacz, Oxford University Press/Clarendon Press, Oxford, pp. 311-327.

(22) Lange, A. (1988b): “Determination of the Radiosonde biases by using Satellite Radiance Measurements.” WMO Instruments and Observing Methods Report No. 33, World Meteorological Organization, Geneva, Switzerland, 1988, pp. 201-206.

(23) Lange, A. (1989): “An Algorithmic Approach for Improving and Controlling the Quality of Upper-Air Data.” WMO Instruments and Observing Methods Report No. 35, World Meteorological Organization, Geneva, Switzerland, 1989.

(24) Lange, A. (1990a): “Real-time Optimum Calibration of large sensor systems by Kalman Filtering.” IEEE PLANS '90 - Position Location and Navigation Symposium Record, March 20-23, 1990, pp. 146-149.

(25) Lange, A. (1990b): “Apparatus and method for calibrating a sensor system.” International Application Published under the Patent Co-operation Treaty (PCT), World Intellectual Property Organization, International Bureau, WO 90/13794, PCT/FI90/00122, 15 November 1990.

(26) Grover Brown, R. and Hwang, P. Y. C. (1992): “Introduction to random signals and Kalman filtering.” John Wiley & Sons, 1992.

(27) Grewal, M. S. and Andrews, A. P. (1993): “Kalman Filtering: Theory and Practice.” Prentice Hall.

(28) Lange, A. (1993): “Method for fast Kalman filtering in large dynamic systems.” International Application Published under the Patent Cooperation Treaty (PCT), World Intellectual Property Organization, International Bureau, WO 93/22625, PCT/FI93/00192, 11 November 1993.

(29) Lange, A. (1994): “A surface-based hybrid upper-air sounding system.” WMO Instruments and Observing Methods Report No. 57, World Meteorological Organization, Geneva, Switzerland, 1994, pp. 175-177.

(30) Julier, S. J., Uhlmann, J. K. and Durrant-Whyte, H. F. (1995): “A New Approach for Filtering Nonlinear Systems.” Proceedings of the 1995 American Control Conference, Seattle, Washington, pp. 1628-1632.

(31) Jatila, E. (1996): “Opening Address.” Proceedings of the Twenty-First Annual Meeting of the International Navigation Association (INA-21), 6-9 August 1996, Helsinki, Finland. International Navigation Association, P.O. Box 2324, Arlington, Virginia 22202-0324, U.S.A., pp. 1-5.

(32) WMO (1996): “Guide to Meteorological Instruments and Methods of Observation.” Sixth edition, WMO-No. 8, World Meteorological Organization, Geneva, Switzerland, 1996.

(33) Lange, A. (1997): “Method for adaptive Kalman filtering in dynamic systems.” International Application Published under the Patent Co-operation Treaty (PCT), World Intellectual Property Organization, International Bureau, WO 97/18442, PCT/FI96/00621, 22 May 1997.

APPENDIX

REGRESSION EQUATIONS FOR HYBRID TRACKING SYSTEMS

The regression equation system for a hybrid tracking radar system that is obtained by combining data based on the non-linear physical dependency equations (1) and (5) can be expressed as follows:

$$\left.\begin{aligned}
h_k &= z_k - z_0 && + c_h && + e_{h,k} \\
\alpha_k &= \arctan((y_k - y_0)/(x_k - x_0)) + \delta(x_k - x_0 < 0)\,\pi && + c_\alpha && + e_{\alpha,k} \\
\varepsilon_k &= \arctan\left((z_k - z_0)/\sqrt{(x_k - x_0)^2 + (y_k - y_0)^2}\right) && + c_\varepsilon && + e_{\varepsilon,k} \\
\rho_k &= \sqrt{(x_k - x_0)^2 + (y_k - y_0)^2 + (z_k - z_0)^2} && + c_\rho && + e_{\rho,k} \\
\varphi_k &= \rho_k + (x_k - x_0)\sin\lambda_\varphi + (y_k - y_0)\cos\lambda_\varphi + \tau_k && + c_\varphi + k\,c_\tau && + e_{\varphi,k}
\end{aligned}\right\}\text{(i)}$$

where h_k, α_k, ε_k, ρ_k and φ_k, for k = 1, 2,…, K, are the measurements of balloon height h, azimuth α, elevation ε, distance ρ and Loran-C pseudo-distance φ, respectively, and where $\delta(x_k - x_0 < 0) = 1$ if $x_k - x_0 < 0$ and otherwise = 0, from an upper-air wind sounding. All equipment is assumed to situate exactly at the station coordinates x_0, y_0 and z_0. The unknown calibration parameters c_h, c_α, c_ε, c_ρ, c_φ and c_τ are assumed to have linear effects on the measurements. The measurement errors $e_{h,k}$, $e_{\alpha,k}$, $e_{\varepsilon,k}$, $e_{\rho,k}$ and $e_{\varphi,k}$ are assumed to be white noise and not to cross-correlate. The balloon positions' coordinates x_k, y_k and z_k, plus the time drift τ_k, are to be estimated. The first position of a balloon is usually known by the station coordinates x_0, y_0 and z_0 plus τ_0 ($\cong$0.0). The balloons must not drift too far, say not more than 50 km away, and the Loran-C transmitter must be distant enough, say 500 km, from the sounding station because no curvature and bearing corrections are made here in these simplified measurement equations.

The Best Linear Unbiased Estimates (BLUE) for iterative Gauss-Newton adjustments are found by differentiating the regression equations (i) with respect to all the four coordinates x_k, y_k, z_k and τ_k of the balloon positions (k = 1, 2,…, K) and all the calibration parameters c_h, c_α, c_ε, c_ρ, c_φ and c_τ. Thus, the linearized regression equation system of the hybrid radar system takes the Canonical Block-Angular form as follows:

$$
\begin{bmatrix} \Delta \boldsymbol{y}_1 \\ \Delta \boldsymbol{y}_2 \\ \vdots \\ \Delta \boldsymbol{y}_K \end{bmatrix} =
\begin{bmatrix} X_1 & & & & G_1 \\ & X_2 & & & G_2 \\ & & \ddots & & \vdots \\ & & & X_K & G_K \end{bmatrix}
\begin{bmatrix} \Delta \mathbf{b}_1 \\ \vdots \\ \Delta \mathbf{b}_K \\ \Delta \mathbf{c} \end{bmatrix} +
\begin{bmatrix} \mathbf{e}_1 \\ \mathbf{e}_2 \\ \vdots \\ \mathbf{e}_K \end{bmatrix} \qquad (6)
$$

wherein the vectors, for k = 1, 2,…, K, are as follows:

$\Delta \boldsymbol{y}_k = [\ \Delta h_k, \Delta \boldsymbol{a}_k, \Delta \boldsymbol{e}_k, \Delta \boldsymbol{r}_k, \Delta \boldsymbol{j}_k]'$
$\mathbf{e}_k\ \ = [\ e_{h,k}, e_{\alpha,k}, e_{\varepsilon,k}, e_{\rho,k}, e_{\varphi,k}]'$
$\Delta \mathbf{b}_k = [\ \Delta x_k, \Delta y_k, \Delta z_k, \Delta \tau_k]'$
$\Delta \mathbf{c}\ = [\ \Delta c_h, \Delta c_\alpha, \Delta c_\varepsilon, \Delta c_\rho, \Delta c_\varphi, \Delta c_\tau]'$

and wherein the submatrices, for k = 1, 2,…, K, are as follows:

$$
X_k = \begin{bmatrix}
0.0 & 0.0 & 1.0 & 0.0 \\
-\sin\alpha_k/\cos\varepsilon_k/\rho_k & \cos\alpha_k/\cos\varepsilon_k/\rho_k & 0.0 & 0.0 \\
-\cos\alpha_k\sin\varepsilon_k/\rho_k & -\sin\alpha_k\sin\varepsilon_k/\rho_k & \cos\varepsilon_k/\rho_k & 0.0 \\
\cos\alpha_k\cos\varepsilon_k & \sin\alpha_k\cos\varepsilon_k & \sin\varepsilon_k & 0.0 \\
\cos\alpha_k\cos\varepsilon_k+\sin\lambda_\varphi & \sin\alpha_k\cos\varepsilon_k+\cos\lambda_\varphi & \sin\varepsilon_k & 1.0
\end{bmatrix}
$$

$$
G_k = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & k
\end{bmatrix}
$$

Numerical solution of the regression equation system (i) often faces singularities caused by missing data or Geometric Dilution of Precision (GDOP). In fact, the last columns of the matrices X_k and G_k are always linearly dependent. This means that only the parameter sums $\tau_k+c_\varphi+kc_\tau$ could probably be estimated. Thus, the linear model (6) of the hybrid radar system (i) is always singular because it exploits only one Loran-C signal. These problems can usually be overcome by regularization. For example, the following tuning error information: $\tau_k - \tau_{k-1} \approx c_\tau$, for k = 1, 2,…, K; could be introduced to the hybrid tracking radar computations as the following ancillary “measurement” equations: $t_k = \tau_k - (\tau_{k-1} + c_\tau) + e_{\tau,k} \equiv 0.0$, for k = 1, 2,…, K, respectively. The presented hybrid radar system always has an accuracy that is at least as good as the respective radar has.

If a distance measurement $\boldsymbol{r}_k$ is missing, then the sum $\tau_k+c_\varphi+kc_\tau$ of the same index k cannot be estimated at all, and the hybrid system performs more like a windfinding theodolite for this balloon position. The hybrid theodolite system converts to a “secondary radar” if the unknown sum $\tau_k+c_\varphi+kc_\tau$ can be estimated by some means. Differential Loran-C and a high-quality time-reference oscillator could be used for that end.

If two Loran-C signals are available and they are not parallel (i.e. $\lambda_\varphi \neq \lambda_\psi \pm n\pi$, for n = 0, 1, 2,…) then the hybrid theodolite system converts to a hybrid “secondary radar” by pure statistical means. Its measurement equations, for k = 1, 2,…, K, are then as follows:

$$
\left.\begin{array}{lll}
x_k = x_k - x_0 & + c_x & + e_{x,k} \\
y_k = y_k - y_0 & + c_y & + e_{y,k} \\
h_k = z_k - z_0 & + c_h & + e_{h,k} \\
\boldsymbol{a}_k = \arctan((y_k-y_0)/(x_k-x_0)) + \delta(x_k-x_0<0)\,\pi & + c_\alpha & + e_{\alpha,k} \\
\boldsymbol{e}_k = \arctan((z_k-z_0)/\sqrt{(x_k - x_0)^2 + (y_k - y_0)^2}) & + c_\varepsilon & + e_{\varepsilon,k} \\
t_k = \tau_k - \tau_0 & & +k\,c_\tau + e_{\tau,k} \\
\boldsymbol{j}_k = \rho_k + (x_k - x_0)\sin\lambda_\varphi + (y_k - y_0)\cos\lambda_\varphi + \tau_k & + c_\varphi & +k\,c_\tau + e_{\varphi,k} \\
\boldsymbol{y}_k = \rho_k + (x_k - x_0)\sin\lambda_\psi + (y_k - y_0)\cos\lambda_\psi + \tau_k & + c_\psi & +k\,c_\tau + e_{\psi,k}
\end{array}\right\} \quad \text{(ii)}
$$

where $x_k = y_k = t_k \equiv 0.0$ and $\rho_k \equiv \sqrt{(x_k - x_0)^2 + (y_k - y_0)^2 + (z_k - z_0)^2}$. The vectors and submatrices of the linearized regression equation system (6) are, for k = 1, 2,…, K, respectively, as follows:

$$\Delta\boldsymbol{y}_k = [0.0, 0.0, \Delta h_k, \Delta\boldsymbol{a}_k, \Delta\boldsymbol{e}_k, 0.0, \Delta\boldsymbol{j}_k, \Delta\boldsymbol{y}_k]'$$
$$\mathbf{e}_k = [e_{x,k}, e_{y,k}, e_{h,k}, e_{\alpha,k}, e_{\varepsilon,k}, e_{\tau,k}, e_{\varphi,k}, e_{\psi,k}]'$$
$$\Delta\mathbf{b}_k = [\Delta x_k, \Delta y_k, \Delta z_k, \Delta\tau_k]'$$
$$\Delta\mathbf{c} = [\Delta c_x, \Delta c_y, \Delta c_h, \Delta c_\alpha, \Delta c_\varepsilon, \Delta c_\varphi, \Delta c_\psi, \Delta c_\tau]'$$

$$
X_k = \begin{bmatrix}
1.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 1.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 1.0 & 0.0 \\
-\sin\alpha_k/\cos\varepsilon_k/\rho_k & \cos\alpha_k/\cos\varepsilon_k/\rho_k & 0.0 & 0.0 \\
-\cos\alpha_k\sin\varepsilon_k/\rho_k & -\sin\alpha_k\sin\varepsilon_k/\rho_k & \cos\varepsilon_k/\rho_k & 0.0 \\
0.0 & 0.0 & 0.0 & 1.0 \\
\cos\alpha_k\cos\varepsilon_k+\sin\lambda_\varphi & \sin\alpha_k\cos\varepsilon_k+\cos\lambda_\varphi & \sin\varepsilon_k & 1.0 \\
\cos\alpha_k\cos\varepsilon_k+\sin\lambda_\psi & \sin\alpha_k\cos\varepsilon_k+\cos\lambda_\psi & \sin\varepsilon_k & 1.0
\end{bmatrix}
$$

$$
G_k = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & k \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & k \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & k
\end{bmatrix}
$$

The launch position coordinates x_1 ($\cong x_0$), y_1 ($\cong y_0$) and z_1 ($\cong z_0$) of a balloon are known with a good absolute accuracy. Such information is necessary for locking phases of the Loran-C signals i.e. for estimating the calibration parameters c_φ and c_ψ. The other balloon coordinates x_k, y_k and z_k, for k = 2, 3,…, K, are not entirely unknown, because no balloon flies faster than e.g. the speed of sound. On the other hand, the frequency stability of a local time-reference oscillator is known from its technical specification. All such ancillary information are to be exploited here in equation system (ii) through using the indicated "measurement" equations of x_k, y_k and t_k in order to avoid ill-conditioned matrix inversions. In fact, a somewhat more effective regularization could be achieved by replacing all x_0, y_0 and τ_0 with x_{k-1}, y_{k-1} and τ_{k-1} for the respective "measurement" equations. However, the remarkably simple analytical structure of the FKF formula (20) would then suffer, though the optimal solution can still be computed fast by a recursive use of Frobenius' formula (22), see Lange (1997).

All these simple regression models presented here can be straight-forwardly generalized to more sophisticated hybrid systems by including measurement equations with varying numbers of different sensors/signals that are used/received at separate locations. The length K of the time-series to be analysed at a time must often be increased in one way or another for improving estimation of the calibration parameters. This may sometimes be achieved by combining measurements made during several sounding times t (t = 1, 2,…). Calibration parameters may, however, vary significantly from sounding to sounding, depending on the calibration stability of equipment. These calibration drifts can be taken into account statistically by formulating the calibration problem as a Kalman Filter.